SOUND GENERATION BY A TOLLMIEN-SCHLICHTING WAVE AT THE END OF A PLATE IN A FLOW

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An important phenomenon in hydrodynamics is the transition from laminar to turbulent flow. It has been established that the transition in a boundary layer on a plate in the presence of low-level freestream perturbations is induced by instability of the laminar flow. The convection (quasilocal) theory of hydrodynamic stability is now well developed; it describes the propagation and downstream buildup of Tollmien–Schlichting (T–S) waves, which also lead to transition.

However, Landau and Lifshitz [1] inform us that the stability problem for steady flow around bodies of finite size has not been theoretically developed to data, and such a mathematical investigation is extremely complicated. In view of the complexity of the problem, it is natural to approach it one step at a time. One such step is to seek out and calculate some kind of upstream transfer of the disturbances. For a perfectly rigid plate we know that disturbances can be transferred upstream only by sound (actually pseudosound). The objective of the present study is to calculate the transmission of sound by a T–S wave at the end of the plate, i.e., to determine the amplitude and structure of the transmitted sound wave.

1. For small Mach numbers M the T–S wavelength $\lambda_1 = 2\pi/\alpha_1$ (α_1 is the x-component of the T–S wave vector; see Fig. 1) is much smaller than the sound wavelength at the same frequency. We can therefore solve the problem in two steps [1], first solving the problem of the incidence of a T–S wave on the end of the plate in the incompressibility approximation and then comparing this solution at distances from the end of the plate much greater than λ_1 with a sound wave. At large Reynolds numbers R the T–S wavelength is much smaller than the length of the plate [2], so that the characteristic length in the x-direction is λ_1. We adopt the velocity of the flow impinging on the plate (freestream velocity) and the thickness of the boundary layer at the end of the plate as our characteristic scales.

The steady-state velocity profile represents boundary layers of unit thickness, which join at the end of the plate, where an inner boundary layer is formed with a thickness of the order of [3] $\delta \sim (X/R)^{1/3}$. The dashed lines in Fig. 1 indicate schematically the boundaries of the upper, lower, and inner boundary layers. The boundary layer above the plate changes with the size of the plate, so that in light of the foregoing considerations its variation with x can be disregarded. The scale y_c of the T–S wave along y is ($u_0(y_c) = \omega/\alpha_1$, where ω is the frequency) on the upper branch of the neutral curve is of the order of [2] $y_c \sim R^{-1/10}$, and the thickness of the inner boundary layer at $x \sim \lambda_1$ is of the order of [2] $\delta \sim (\alpha_1 R)^{-1/3} \sim R^{-3/10}$. We therefore have $\delta/y_c \sim R^{-1/5}$ at $x \sim \lambda_1$. In this situation it is natural to replace the inner boundary at large R by a surface of discontinuity, where matching conditions must be established, and the steady-state velocity profile is assumed to be independent of the coordinate x. The required matching conditions are continuity of the velocity, pressure, and the derivative of the tangential component of the velocity along the normal to the discontinuity surface.

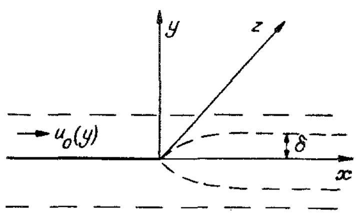

Fig. 1

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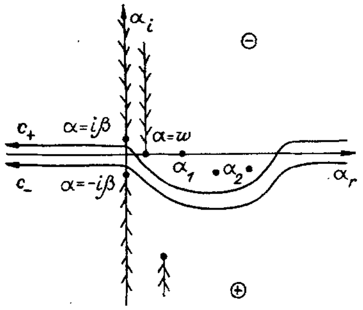

Fig. 2

We write the velocity vector in the form of a sum $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1$, where $\mathbf{v}_1$ is a small perturbation, and the steady flow velocity is

$$\mathbf{v}_0 = (u_0(y), 0, 0). \tag{1.1}$$

In this notation, according to the foregoing discussion, we formulate the first step of the problem as follows: for the equations

$$\frac{\partial \mathbf{v}_1}{\partial t} + u_0 \frac{\partial \mathbf{v}_1}{\partial x} + u_0'(\mathbf{v}_1 \cdot \mathbf{j})\,\mathbf{i} = -\operatorname{grad} p + \frac{1}{\mathrm{R}} \Delta \mathbf{v}_1, \operatorname{div} \mathbf{v}_1 = 0 \tag{1.2}$$

find a solution

$$\mathbf{v}_1 = \begin{cases} \exp[-i(\omega t - \beta z)]\,(\mathbf{v}_{\text{T-S}}(y)\exp(i\alpha_1 x) + \mathbf{v}_-), & x < 0, \\ \exp[-i(\omega t - \beta z)]\,\mathbf{v}_+, & x > 0, \end{cases} \tag{1.3}$$

that satisfies the matching conditions

$$[\mathbf{v}_1] = \left[\frac{\partial \mathbf{v}_1}{\partial y}\right] = [p] = 0, \quad [f] = f(x, +0) - f(x, -0), \ x > 0 \tag{1.4}$$

and the boundary conditions

$$\mathbf{v}_1 = 0, \ y = 0, \ x < 0; \ \mathbf{v}_1 \to 0, y \to \pm\infty \tag{1.5}$$

($\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the x and y axes, and i denotes imaginary unity). In Eq. (1.3) $\mathbf{v}_{\text{T-S}}(y)\exp(i\alpha_1 x + i\beta z)$ is a T–S wave of specified amplitude incident on the end of the plate at $y \geq 0$, $\mathbf{v}_-$ is a linear combination of waves traveling to the left from x = 0, $\mathbf{v}_+$ is a combination of waves propagating to the right from x = 0 (the significance of dividing the waves into two groups is clarified in [4]), and β is the z-component of the T–S wave vector; it is a small quantity

$$\beta < \mathrm{M}\omega, \tag{1.6}$$

so that the generated sound wave does not exist otherwise.

Since Eq. (1.2) and conditions (1.4) and (1.5) are symmetric about y = 0, $\mathbf{v}_1$ can be represented as the sum of a symmetric solution and an antisymmetric solution. These solutions are sought independently.

2. We write the antisymmetric solution as the sum

$$\mathbf{v}_1 = \exp[-i(\omega t - \beta z)]\left(\frac{1}{2}\mathbf{v}_{\text{T-S}}\exp(i\alpha_1 x) + \mathbf{v}_2\right), \mathbf{v}_2 = (u, v, w). \tag{2.1}$$

Eliminating the time and the coordinate z from Eq. (1.2) by means of Eq. (2.1) and then applying the generalized Fourier transform with respect to x [5], we obtain the following relations for the Fourier transform V [2]:

$$L^2 V = i\alpha\mathrm{R}\left[(u_0 - c)L - u_0''\right]V, L = \frac{d^2}{dy^2} - \gamma^2, \gamma^2 = \alpha^2 + \beta^2, c = \frac{\omega}{\alpha}; \tag{2.2}$$

$$V_- = V'_- = V'_+ = V'''_+ + i\alpha R u'_0 V_+ + i v'''_{T\text{-}S} /(2\sqrt{2\pi}(\alpha - \alpha_1)) = 0, \ y = 0; \quad V \to 0, \ y \to \pm\infty, \tag{2.3}$$

where

$$V = V_- + V_+, V_- = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} v \exp(-i\alpha x)\, dx, \tag{2.4}$$

$$V_+ = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} v \exp(-i\alpha x)\, dx.$$

Conditions (2.3) are obtained as follows: $V_-(0) = 0$ follows from Eq. (2.4) and condition (1.5), $V'_-(0) = 0$ follows from Eqs. (2.4), (1.5) and the equation of continuity, $V_+'(0) = 0$ follows from Eq. (2.4) and the symmetry of the sought-after solution, and the final equation (2.3) is a consequence of the discontinuity of u_0' at $y = 0$, conditions (1.4), and the symmetry of the solution.

Problem (2.2)-(2.4) is solved by the Wiener–Hopf method [6]. The domains of analyticity and singularities of V_- and V_+ are shown schematically in Fig. 2; the domain of analyticity of V_- lies above the contour c_-, and that of V_+ is below the contour c_+. The singularities of V_- and V_+ are determined by the form of the disturbances propagating to the left and to the right of $x = 0$. They are four branch points given by the equations $\gamma = 0$ and $\alpha - \omega - i\gamma^2/R = 0$, along with isolated singularities, which are obtained as the corresponding part of the discrete spectrum of the Orr–Sommerfeld operator (2.2), (2.3). It is necessary to choose either the first two or the last two conditions in order to find the spectrum in Eqs. (2.3) at $y = 0$. In Fig. 2 α_r and α_i are the real and imaginary parts of α. Assuming that Eq. (2.2) has a solution which decays in the limit $y \to \infty$, we write it in the form

$$V = c_1\varphi_1 + c_2\varphi_2 \tag{2.5}$$

(φ_1 is a decaying nonviscous solution, and φ_2 is a decaying viscous solution [2]). Using a procedure described in [6] [eliminating c_1 and c_2 from Eqs. (2.3)-(2.5)], we obtain

$$V'''_- - \frac{a}{\alpha - \alpha_1} = V_+ F, a = i\gamma^2(\alpha_1) R p_{T\text{-}S} /(2\sqrt{2\pi}), \ F = \left[i\alpha R u'_0 + \frac{\varphi'''_1\varphi'_2 - \varphi'''_2\varphi'_1}{\varphi_1\varphi'_2 - \varphi'_1\varphi_2} \right]_{y=0} \tag{2.6}$$

($p_{T\text{-}S}$ is the pressure in the T–S wave at the end of the plate at $y = 0$). Making use of the behavior of φ_1 and φ_2, we find

$$\lim_{\alpha_r \to \pm\infty} (F/(2\gamma^3)) = 1.$$

We can therefore factor F, following [6], in the form

$$F = 2\gamma_-^3 \gamma_+^3 K_+/K_-, \gamma_- = \sqrt{\alpha + i\beta}, \gamma_+ = \sqrt{\alpha - i\beta},$$
$$K_- = \exp\left(\frac{1}{2\pi i} \int_{c_-} \frac{\ln(F/2\gamma^3)}{t - \alpha} dt \right), \ K_+ = \exp\left(\frac{1}{2\pi i} \int_{c_+} \frac{\ln(F/2\gamma^3)}{t - \alpha} dt \right). \tag{2.7}$$

Repeating the procedure of [6], we reduce Eq. (2.6) to the form

$$\frac{V'''_- K_-}{\gamma_-^3} - \frac{a}{\alpha - \alpha_1}\left(\frac{K_-}{\gamma_-^3} - \frac{K_-(\alpha_1)}{\gamma_-^3(\alpha_1)} \right) = 2\gamma_+^3 K_+ V_+ + \frac{a}{\alpha - \alpha_1} \frac{K_-(\alpha_1)}{\gamma_-^3(\alpha_1)} = J(\alpha). \tag{2.8}$$

The left-hand side of this equation is analytic above c_- (Fig. 2), and the right-hand side is analytic below c_+. Consequently, the function $J(\alpha)$ represented by Eq. (2.8) is analytic in the entire plane. It has been shown [7] that the stream function behaves as follows at short distances from the end of the plate:

$$\Psi \sim r^{3/2}\left(\cos\frac{3\theta}{2} + 3\cos\frac{\theta}{2} \right), r = \sqrt{x^2 + y^2}, \theta = 2 \operatorname{arctg} \frac{y}{r - x}. \tag{2.9}$$

Accordingly, $v(x, 0) \sim \sqrt{x}$ at $x > 0$. Then $V_+(0) \sim \alpha^{-3/2}$ in the limit $\alpha \to \infty$, $\alpha_i < 0$ [8]; K_- and K_+ tend to unity in the limit $\alpha \to \infty$ in their domains of analyticity. It follows from these estimates and Eq. (2.8) that $J(\alpha)$ is bounded in the half-plane $\alpha_i < 0$. The function $J(\alpha)$ is readily estimated for $\alpha_i < 0$. By virtue of relation (2.9), the pressure p admits the estimate $p(x, 0) \sim x^{-1/2}$, $x < 0$, at $r \ll 1$. As a result, exploiting the symmetry of the problem, we obtain the Fourier transform of the pressure

$$P(0) = P_-(0) \sim \alpha^{-1/2},\ \alpha \to \infty,\ \alpha_i > 0.$$

Next, expressing $V'(0)$ and $P(0)$ in terms of φ_1 and φ_2 by means of Eq. (2.5) and invoking conditions (2.3), we can estimate the behavior of the constants in Eqs. (2.5) in the same domain $\alpha_i > 0$. Now, writing $V'''(0)$ in the form (2.5) and making use of conditions (2.3), we have

$$V_-'''(0) \sim \alpha^{3/2},\ \alpha \to \infty,\ \alpha_i > 0.$$

It follows from this result and from Eq. (2.8) that $J(\alpha)$ is bounded for $\alpha_i > 0$. Consequently $J(\alpha)$ is bounded in the entire plane and is therefore equal to a constant. Expressing $V_-'''(0)$ from Eq. (2.8) and simplifying the result, we obtain

$$V_-''' = \frac{a}{\alpha - \alpha_1} - b,\ b = \frac{a\gamma_-}{\gamma_-(\alpha_1)} \exp\left[\frac{\alpha - \alpha_1}{2\pi i} I(\alpha)\right]\left(\frac{1}{\alpha - \alpha_1} - \text{const}\right), \tag{2.10}$$

$$I(\alpha) = \int\limits_{c_-} \frac{\ln(F/\gamma)}{(t - \alpha_1)(t - \alpha)} dt.$$

Writing V' and $V''' + i\alpha R u_0' V$ at $y = 0$ in the form (2.5) and invoking conditions (2.3), we obtain

$$c_1\varphi_1' + c_2\varphi_2' = 0,\ c_1(\varphi_1''' + i\alpha R u_0'\varphi_1) + c_2(\varphi_2''' + i\alpha R u_0'\varphi_2) = -b.$$

We find c_1 and c_2 from this system, and for the Fourier transform of the potential

$$\Phi = \int\limits_{\infty}^{y} V\, dy$$

we obtain

$$\Phi = \frac{b\exp(-\gamma y)}{\gamma}\left[\frac{\varphi_2'}{\varphi_2'(\varphi_1''' + i\alpha R u_0'\varphi_1) - \varphi_1'(\varphi_2''' + i\alpha R u_0'\varphi_2)}\right]_{y=0},\ y > 1. \tag{2.11}$$

The potential of the perturbations specifying the sound wave is determined at large distances ($r \gg \lambda_1$) by the behavior of Φ in the neighborhood of $\gamma = 0$. We must therefore investigate the behavior of the factors in Eq. (2.11) in the limit $\gamma \to 0$. Using Eq. (2.2) and the attenuation conditions for φ_1, and φ_2, we write the expression for F in the form

$$F = \gamma^2 R\left[\frac{\int\limits_0^\infty\left(i\alpha u_0 - i\omega + \frac{\gamma^2}{R}\right)\varphi_1\, dy\,\varphi_2' - \int\limits_0^\infty\left(i\alpha u_0 - i\omega + \frac{\gamma^2}{R}\right)\varphi_2\, dy\,\varphi_1'}{\varphi_1\varphi_2' - \varphi_1'\varphi_2}\right]_{y=0}. \tag{2.12}$$

The numerator in Eq. (2.12) is proportional to γ in the limit $\gamma \to 0$, and it is easily verified by the direct substitution of φ_1 and φ_2 that the denominator does not vanish in this limit [2]. Consequently, F/γ has a nonzero finite value in the limit $\gamma \to 0$, and so $I(-i\beta)$ exists in Eq. (2.10). To within a multiplicative factor, the denominator of expression (2.11) coincides with the numerator of (2.12), which is readily calculated in the limit $\gamma \to 0$. Carrying out the calculations for Φ in the neighborhood of $\gamma = 0$, we obtain

$$\Phi \approx \frac{ia\left(\frac{1}{\alpha - \alpha_1} - \text{const}\right)}{\omega R\gamma_+^2\gamma_-\gamma_-(\alpha_1)} \exp\left[\frac{\alpha_1 + i\beta}{2\pi i} I(-i\beta) - \gamma y\right],\ y > 1,\quad \gamma \to 0. \tag{2.13}$$

The constant in Eq. (2.13) is chosen for $\beta = 0$. In order for the potential to decay in the limit $\beta \to 0$, $r \to \infty$, the constant must be equal to $-1/\alpha_1$. For $\beta \neq 0$ its value differs from $1/\alpha_1$ by a small term of the order of β ($\beta < M\omega$). Accordingly,

$$\Phi \approx \frac{a_1}{\gamma_-} \exp\left[\frac{\alpha_1 + i\beta}{2\pi i} I(-i\beta) - \gamma y\right], a_1 = -ia/(\omega R \alpha_1^{5/2}), y > 1, \gamma \to 0. \tag{2.14}$$

3. We calculate the limits

$$\frac{\alpha_1 + i\beta}{2\pi i} I(-i\beta) \tag{3.1}$$

as the neutral curve tends to infinity along the upper branch [2]:

$$R \to \infty, \omega \sim R^{-1/5}, \alpha_1 \approx \sqrt{u_0' \omega}. \tag{3.2}$$

We first transform $I(-i\beta)$. Using the expression for F (2.12) and the asymptotic representations for φ_1 and φ_2 [2], we have

$$\begin{gathered} F/\gamma = i\omega R f'(0)\Pi/(\gamma f(0)),\ \Pi = 1 + 0(1/\sqrt{\omega R}), \\ f = 1 - \gamma(1-c)^2 \int_1^y (u_0 - c)^{-2}\, dy + \gamma^2 \int_1^y (u_0 - c)^{-2} \int_1^y (u_0 - c)^2\, dy\, dy - \ldots \end{gathered} \tag{3.3}$$

The asymptotic expansions of φ_1 and φ_2 given in [2] can be used on c_-, since c_- lies in the domain $\alpha_i < 0$ and $c_i = \mathrm{Im}(\omega/\alpha) > 0$. As for the small neighborhood of the point $\alpha = \omega$ where the given asymptotic expansions are invalid, we keep the contour of integration far from this point in making the limiting transition (3.2). We introduce D by means of the equation

$$\frac{F}{\gamma} = -\frac{iR(\alpha - \omega)^2 \Pi(-i\beta) D}{\omega}, \tag{3.4}$$

It is readily verified that $D(-i\beta) = 1$. If F/γ in the form (3.4) is substituted into the expression for I (2.10), I decomposes into a two-term sum, and we close the first contour (Fig. 3a) through the lower half-plane, so that it is equal to zero. In regard to the term containing ln D, here we go from the contour c_- to the contour c_-' (Fig. 3a). In this case the integral along c_- is equal to the sum of the integral along c_-' and the residue at the point $t = \alpha$. The residue tends to zero in the limit $\alpha \to -i\beta$ [$D(-i\beta) = 1$], and expression (3.1) acquires the form

$$\frac{1 + i\beta/\alpha_1}{2\pi i} \int_{c_-'} \frac{\ln D(t')\, dt'}{(t'-1)(t' + i\beta/\alpha_1)}, \quad t = \alpha_1 t'. \tag{3.5}$$

Analyzing the behavior of $D(t')$ on the basis of Eqs. (3.3) and (3.4) in the limit $R \to \infty$ (3.2), we can verify that $D(t')$ tends uniformly to the limiting function on any finite arc that emanates from the origin, does not contain $t' = 0$, and lies in the interval $t_i' \leq 0$:

$$D(t') \to 1 - \gamma t', \quad \gamma = \sqrt{t_+'}\sqrt{t_-'}.$$

The cuts $\sqrt{t_+'}$ and $\sqrt{t_-'}$ are shown in Fig. 3b. It is also readily shown that the integral (3.5) converges uniformly with respect to R in the limit $R \to \infty$ (3.2). It follows from these considerations that the quantity (3.1) obtained in the limit $R \to \infty$ (3.2) is equal to the integral of the limiting function

$$\lim_{R \to \infty} \frac{\alpha_1 + i\beta}{2\pi i} I(-i\beta) = \frac{1}{2\pi i} \int_{c_-} \frac{\ln(1 - \gamma t')\, dt'}{(1-t')\, t'}. \tag{3.6}$$

We deform the contour c_- into the contour Γ (Fig. 3b) and reduce the integral (3.6) to the tabulated form [8]

$$\frac{1}{2\pi i} \int_{c_-} \frac{\ln(1 - \gamma t')}{(1-t')\, t'}\, dt' = \frac{1}{2\pi i} \left[i \int_0^\infty \frac{\ln(1 + x^2)\, dx}{1 + x^2} + \right.$$

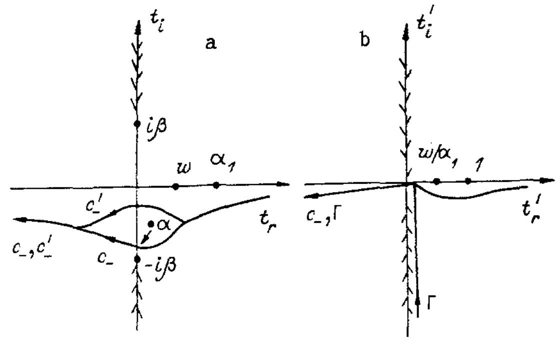

Fig. 3

$$+\int_0^\infty \frac{\ln(1+x^2)}{x}\left(\frac{1}{1+x}-\frac{1}{1+x^2}\right)dx\Bigg] = \ln\sqrt{2}+\frac{i\pi}{8}. \tag{3.7}$$

Now, substituting Eq. (3.7) into (2.14) and making use of Eq. (2.6), we obtain the following expression for the Fourier transform of the potential in the limit R → ∞ in the neighborhood $\gamma \to 0$:

$$\Phi \approx \frac{p_{\text{T-S}}}{2\omega\sqrt{\pi\alpha_1(\alpha+i\omega)}}\exp\left(\frac{i\pi}{8}-\gamma y\right), y>1. \tag{3.8}$$

For the potential at large distances ($(M\omega)^{-1} \gg r \gg 1$) we find [8]

$$\varphi = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Phi\exp(i\alpha x)\,dx = \frac{p_{\text{T-S}}\exp\left(-\frac{i\pi}{8}\right)}{\omega\pi\sqrt{2\alpha_1}}\int_{-\infty}^{\infty}\frac{\exp(-ty)\cos\left(tx+\frac{\pi}{4}\right)}{\sqrt{t}}dt = \tag{3.9}$$
$$= \frac{p_{\text{T-S}}\exp\left(-\frac{i\pi}{8}\right)}{\omega\sqrt{2\pi\alpha_1}}\frac{\sin(\theta/2)}{\sqrt{r}}, \quad y>1$$

(θ and r are given by Eqs. (2.9)).

4. We now examine the behavior of the acoustic potential at $r \gg 1$, seeking it in the form

$$\varphi = \exp[-i(\omega t-\beta z)]\sin(\theta/2)g(r). \tag{4.1}$$

Substituting Eq. (4.1) into the sound propagation equation

$$\Delta\varphi = M^2\frac{\partial^2\varphi}{\partial t^2}, \tag{4.2}$$

we have

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dg}{dr}\right)+\left(k^2-\frac{1}{4r^2}\right)g=0,\ k^2 = M^2\omega^2-\beta^2. \tag{4.3}$$

The solution of Eq. (4.3) for a wave traveling away from the origin is written in the form

$$g = C\frac{\exp(ikr)}{\sqrt{r}}. \tag{4.4}$$

Comparing Eqs. (4.4) and (3.9) in the interval $(M\omega)^{-1} \gg r \gg 1$, at $r \gg 1$ we find

$$\varphi = \frac{p_{\text{T-S}}}{\omega\sqrt{2\pi}}\frac{\sin(\theta/2)}{\sqrt{r\alpha_1}}\exp\left[-i\left(\omega t-\beta z-kr+\frac{\pi}{8}\right)\right]. \tag{4.5}$$

The pressure in the sound wave $p = -\partial\varphi/\partial t$ acquires the form

$$p = -\frac{p_{\text{T-S}}}{\sqrt{2\pi}}\frac{\sin(\theta/2)}{\sqrt{r\alpha_1}}\exp\left[-i\left(\omega t - \beta z - kr + \frac{5\pi}{8}\right)\right].$$

Let L be the length of the plate. The potential (4.5) represents the generated sound wave for $Lk \gg 1$. Indeed, the amplitude of the sound wave arriving from the end of the plate has the following relative order of magnitude at the forward edge:

$$\frac{1}{\sqrt{L\alpha_1}} \sim \frac{1}{\sqrt{Lk}}\sqrt{\frac{k}{\alpha_1}}.$$

We adopt $r\alpha_1 = 1$ conditionally as the origin. Consequently, sound generated at the forward edge cannot significantly alter the sound field 4.5). For

$$Lk \ll 1 \tag{4.6}$$

the forward edge of the plate lies in the zone

$$k^{-1} \gg r \gg 1\,, \tag{4.7}$$

and the potential (3.9) cannot be used for matching with the sound wave. The potential in the domain (4.7), subject to condition (4.6), must be sought as the solution of the following problem: find a solution of the Laplace equations subject to the conditions

$$\varphi = 0,\ y = 0,\ x < -L;\ \partial\varphi/\partial y = 0,\ y = 0,\ -L < x < 0;$$
$$\varphi = 0,\ y = 0,\ x > 0.$$

In the vicinity of $r = 0$ the potential φ has a singularity of the form (3.9) and is bounded in the remaining space. The solution of this problem is unique [9] and has the form

$$\varphi = -\frac{p_{\text{T-S}}\exp\left(-\frac{i\pi}{8}\right)}{\omega\sqrt{2\pi\alpha_1 L}}\operatorname{Im}\left(\sqrt{\frac{W+L}{W}}\right),\quad W = x + iy.$$

At $r \gg L$ we have

$$\varphi \approx \frac{p_{\text{T-S}}\exp\left(-\frac{i\pi}{8}\right)}{2\omega\sqrt{2\pi\alpha_1/L}}\frac{\sin(\theta)}{r}. \tag{4.8}$$

The acoustic potential, which goes over to (4.8) at $k^{-1} \gg r \gg L$ is written as follows at distances $kr \gg 1$ [1]:

$$\varphi = \frac{p_{\text{T-S}}\sqrt{kL}}{4\omega}\frac{\sin(\theta)}{\sqrt{r\alpha_1}}\exp\left[-i\left(\omega t - \beta z - kr + \frac{3\pi}{8}\right)\right], \tag{4.9}$$

and the corresponding pressure is

$$p = -p_{\text{T-S}}\frac{\sqrt{kL}}{4}\frac{\sin(\theta)}{\sqrt{r\alpha_1}}\exp\left[-i\left(\omega t - \beta z - kr + \frac{7\pi}{8}\right)\right].$$

5. The results discussed in Secs. 2-4 represent the solution of the antisymmetric problem. For the solution symmetric in u, the amplitude of the incident T-S wave is the same as for the antisymmetric solution, and the conditions at $y = 0$ have the form

$$v = \partial v/\partial y = 0,\ x = 0;\ v = \partial^2 v/\partial y^2 = 0,\ x > 0. \tag{5.1}$$

Repeating the calculations of Sec. 2, we obtain the Fourier transform of the potential in the limit $\gamma \to 0$:

$$\Phi \approx \frac{p_{\text{T-S}}\,\alpha}{2\omega\gamma\sqrt{-i2\pi\omega R}}\exp\left[\frac{1}{2\pi i}\int\limits_{c_-}\frac{\ln(F(t\alpha_1)/F(0))\,dt}{(t-1)\,t} - \gamma y\right],\ y > 1,\ \beta = 0,$$

$$F(\alpha)=\left[\frac{\varphi_1\varphi_2''-\varphi_2\varphi_1''}{\varphi_1\varphi_2'-\varphi_2\varphi_1'}\right]_{y=0}. \tag{5.2}$$

The integral in expression (5.2) tends to zero in the limit $R \to \infty$, since $F(t\alpha_1)/F(0) \to 1$ on any finite part of c_- (Fig. 3b). This can be verified on the basis of asymptotic expansions of φ_1 and φ_2 [2]. We write expression (5.3) as follows in the limit $\gamma \to 0$, $R \to \infty$:

$$\Phi \simeq \frac{p_{\text{T-S}}\alpha}{2\omega\gamma\sqrt{-i2\pi\omega R}}\exp(-\gamma y),\ y>1,\ \beta=0. \tag{5.3}$$

It follows from a comparison of Eqs. (3.8) and (5.3) that the contribution of the symmetric solution to the generation of sound can be disregarded.

In fact, the boundary conditions (5.1) for the symmetric solution are reduced to the following conditions in the nonviscous limit:

$$v=0,\ y=0,\ x<0;\quad v=0,\ y=0,\ x>0.$$

The T–S wave is not scattered under such conditions.

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